Int. J. Heat Mass Transfer. Vol. 12. pp. 771–789. Pergamon Press 1969. Printed in Great Britain

THE TURBULENT BOUNDARY LAYER ON A POROUS PLATE: EXPERIMENTAL SKIN FRICTION WITH VARIABLE INJECTION AND SUCTION

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(*Received* 29 *April* 1968 *and in revised form* 5 *August* 1968)

Abstract—Experimental skin friction results from constant free-stream velocity boundary layers are reported for a variety of constant and slowly varying injection and suction wall conditions. A description is given of the flow characteristics of these air experiments.

The uniform injection results are in good agreement with the results of Kendall and the Stevenson, Rotta, and Kinney results from the Mickley–Davis data. For all turbulent flows examined, $C_f/2$ is found to be a a function of local Re_θ and B. The friction factor ratio $C_f/C_{fo}|_{Re_\theta}$ is found to be a function of B alone, and is given as an empirical function of B.

Of seven theories examined, the theories of Rubesin and of Torii *et al.* are in best agreement with all of the results when considered on a local Re_θ and B basis. A simple calculation method of $C_f/2$ vs. Re_x is suggested for slowly varying $V_w(X)$.

NOMENCLATURE

a, d, constants in equation (4);

B, $= \dot{m}''/G(C_f/2)$, blowing parameter;

b, $= \dot{m}''/G(C_{f_0}/2)$, blowing parameter;

$C_f/2$, friction factor;

c, constant [lbm/s-ft^2];

exp, base of natural logarithms;

f, g, denote functions;

G, $= (\rho U)_\infty$[lbm/s-ft^2];

k_w, surface roughness size [in.];

$\dot{m}''$, $= (\rho V)_w$ [lbm/s-ft^2];

n, constant in equation (16);

Pr, Prandtl number;

Re_x, position Reynolds number, $= U_\infty X/\nu_\infty$;

Re_θ, momentum thickness Reynolds number, $= U_\infty \theta/\nu_\infty$;

St, Stanton number;

U velocity in the main-stream direction [ft/s];

U^+, dimensionless velocity $= U/U_\tau$;

U_τ $= \sqrt{(\tau_w g_c/\rho)}$ [ft/s];

V, velocity perpendicular to the surface [ft/s];

V_w/U_∞, velocity ratio;

X, distance along the plate in the flow direction [ft];

y, perpendicular dlstance from the surface [ft];

y^+, dimensionless distance, $= yU_\tau/\nu$;

Greek symbols

δ, boundary-layer thickness [ft];

θ, momentum thickness [ft],

$$= \int_0^\infty \frac{\rho U}{\rho_\infty U_\infty}\left(1 - \frac{U}{U_\infty}\right) dy;$$

λ, dummy variable;

μ, viscosity [lbm/s-ft];

ν, kinematic viscosity [ft^2/s];

ρ, density [lbm/ft^3];

τ, shear stress [lbf/ft^2].

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Subscripts

a,	denotes value at outer edge of sublayer;
crit,	denotes condition when $C_f/2 = 0$;
0,	refers to unblown condition;
w,	denotes wall condition;
∞,	denotes free-stream condition.

1. INTRODUCTION

THE DEVELOPMENT of accurate methods for predicting the behaviour of turbulent boundary layers requires an adequate experimental base. In particular, reliable data regarding skin friction, heat transfer, velocity and temperature profiles are needed over a wide range of conditions with blowing and suction. These measurements are difficult to obtain since the problem includes all of the usual difficulties inherent in turbulent boundary layer studies plus a host of new problems associated with the blowing. The absence of a well verified body of data makes it difficult to judge the validity of new data as it appears in the literature: no base line has yet been agreed upon. It is necessary, therefore to fall back on basic tests of data for self-consistency (mass, momentum, and energy conservation), and on judgements of the apparatus used. Data which are not self-consistent are clearly inadmissible. Apparatus proposed for studies with blowing or suction should be tested by running unblown test cases and reproducing acceptable data.

A review of the existing data on the turbulent boundary layer with blowing and suction shows a need for further work, even in the simplest cases. The present paper restricts its attention to the skin friction problem of two-dimensional, constant property, constant freestream-static-pressure flow over a smooth flat plate, heat transfer and structure being covered in separate publications [1–3].

1.1 *Review of previous experimental works*

One of the pioneering works in this area was the extensive study reported in 1954 by Mickley, Ross, Squyers, and Stewart [4]. Skin friction results of this study were criticized in 1957 by Mickley and Davis [5] who reported that mechanical difficulties with the apparatus had introduced unsuspected errors into the earlier study.

Davis [6], Kendall [7], Butensky [8], and Smith [9] used this same apparatus, in various configurations, to produce additional data for constant velocity flows with uniform blowing. Davis and Kendall each reported significant non-uniformity in the transpiration flow rates, again due to mechanical problems, while Kendall documented other characteristics of the apparatus which required careful handling. Butensky attempted to confirm Smith's data, but was not successful, attributing the difference to changes in the apparatus. Fraser [10] pointed out that his results did not reproduce the accepted "law of the wall" in tests with no blowing and concluded that his flow was not characteristic of an ordinary two-dimensional turbulent boundary layer.

Of all the preceding studies only those of Kendall and Mickley-Davis were able to reproduce the accepted results for an unblown flat plate (velocity profile and skin friction). Their blown results are summarized in Fig. 1. In this figure, the symbol $C_{f_0}/2$ refers to the value of

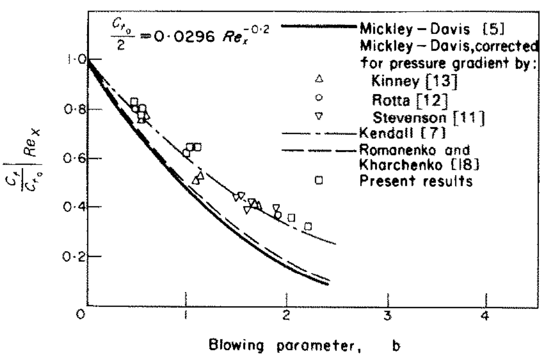

FIG. 1. Comparison of experimental data, $Re_x \approx 10^6$, uniform injection.

friction coefficient which would have been measured at the same X-Reynolds number had there been no blowing. The Mickley–Davis results, as originally reported, lie considerably below those of Kendall. Stevenson [11], Rotta

[12], and Kinney [13] examined the Mickley–Davis results and corrected them for the effects of the reported pressure gradient (which Mickley and Davis had neglected) by means of the momentum integral equation. The corrected results shown in Fig. 1 were obtained for several different test runs using a continuous longitudinal pressure gradient distribution for each run. These results are found to agree well with Kendall's values. Hartnett *et al.* [14] also suggested that the Mickley–Davis results were low, as originally presented, on the basis of comparison with Rubesin's theory [15] which had checked well with compressible flow results.

Data from two other sources were considered. McQuaid [16] reported $C_f/2$ values deduced from measurements of momentum thickness at successive stations along a porous plastic plate. Examination of his data in $C_f/2$ vs. Re_θ coordinates, as proposed by Rotta [17], reveals an apparent velocity dependence which was not expected [1]. Rotta proposed that the following should apply:

$$\frac{C_f}{2} = g\left(Re_\theta, \frac{V_w}{U_\infty}\right). \tag{1}$$

The existence of a velocity dependence in McQuaid's data, even though inside the quoted uncertainty band, suggests that the structure of the apparatus might, in some way, have affected the results.

Romanenko and Kharchenko [18] presented results for $(C_f/C_{f0}|_{Re_x})$ in the form used in Fig. 1 but did not identify which of two methods were used to determine skin friction. There is, additionally, some question as to whether or not the small size of the porous region (6 × 30 cm in the flow direction) is adequate to describe a "uniform blowing" case in view of the thickness of the boundary layer with strong blowing.

Several investigators have studied the case with uniform suction [19–22]. The suction term is additive in the momentum integral equation, hence skin friction can be determined more accurately for suction than for blowing. In addition, the effects of three dimensional flow are less important, due to the high friction, again favoring these studies.

Suction tends to decrease the turbulent disturbances in the boundary layer. Strong suction, $V_w/U_\infty \cong -0{\cdot}01$, results in a laminar layer having many characteristics of an asymptotic layer. For very small values of suction, the boundary layer remains turbulent in character, closely resembling the unsucked layer. Dutton [22] found that in the intermediate range the development of a turbulent boundary layer with uniform suction depends on two principal factors: the surface condition, and the state of the boundary layer at the point where suction begins.

1.2. *Objectives of the present work*

The motivation for the present work can be summarized as follows. It is necessary to have a broad base of well documented skin friction results, taken from a qualified apparatus, in order to extend turbulent boundary layer theory to cases with blowing and suction. Review of the present literature shows only a limited amount of data which is of unchallenged reliability and, in no case, has a single apparatus been used over a wide range of blowing and sucking conditions.

In broad terms, the objectives of this paper are:

(1) To present and document experimental skin friction results for constant free-stream velocity boundary layers for a variety of constant and slowly varying injection and suction wall conditions. Here "slowly varying" is defined by the inequality

$$\left|\frac{\mathrm{d}V_w}{\mathrm{d}X}\right| \ll \frac{\partial U}{\mathrm{d}y}(X, 0).$$

(The velocity profile data and correlations associated with these skin friction results are tabulated and discussed in [1].)

(2) To compare these results with existing data and theories and to provide a simple

calculation scheme of $C_f/2$ vs. Re_x for constant and slowly varying injection or suction cases.

2. EXPERIMENTAL APPARATUS

The Stanford Heat and Mass Transfer Apparatus, as described in detail by Moffat and Kays [2, 23], was used in these experiments. This apparatus consists of a 24-plate porous surface, 8 ft long and 18 in. wide. The plates form the lower surface of a test duct of rectangular cross section, 20 in. wide and 6 in. high at the inlet end of the duct. A boundary layer trip is located just upstream of the leading edge of the porous surface. The upper surface is adjustable to achieve a uniform velocity along the duct, regardless of the distribution of the blowing or suction along the porous surface. The $\frac{1}{4}$ in. thick sintered bronze plates are smooth to the touch and are uniform in porosity within 6 per cent in the 6-in. span centered on the test duct centerline, where velocity profiles are taken. The flow through each plate is individually controlled. Separate mainstream and transpiration blowers provide the system with air that had been filtered, while heat exchangers are used to control air temperature.

All measurement of gas temperatures were made with iron–constantan thermocouples, described in detail by Moffat and Kays [23]. Calibrated rotameters were used to measure injection and suction flow rates.

Mean velocity profiles were measured with stagnation pressure probes and manual traversing equipment. The dynamic pressures were measured with calibrated inclined manometers. The probes were attached to traversing mechanisms fastened to a rigid support frame. These spring-loaded micrometer-driven mechanisms provided for the change and measurement of probe distance from the test wall.

The probes used for boundary layer surveys have a flattened mouth 0·010 × 0·035 in. formed from 0·025-in. o.d., 0·0025-in. wall thickness tubing. When in operating position the tangent to the tip formed an included angle of about 3° with the test surface. Laboratory wind tunnel tests revealed that a yaw angle or a pitch angle of at least $\pm 10°$ produced no detectable change in indicated maximum dynamic pressure.

Static pressure taps are located on 12 in. centres next to plate segments 2, 5, 8, 11, 14, 17, 20, and 23 on one side wall of the test section. At each longitudinal position the free-stream static pressure sensed by a pitot-static Prandtl probe was found equal to that sensed by the side wall tap. All present data were taken using the side wall taps.

3. QUALIFICATION OF THE APPARATUS

The fluid dynamic characteristics of the apparatus were examined in view of the requirements for the ideal flow model: steady, two-dimensional, constant property, constant free-stream velocity, turbulent flow over a smooth uniformly permeable flat plate.

3.1. *The impermeable flat plate data*

Since the special case with $V_w = 0$ has been previously examined in detail, the apparatus must produce acceptable data in order to be qualified for use in experiments with $V_w \neq 0$. The following summarizes the results of the qualification tests for this special case:

(a) Friction factors obtained by the momentum integral equation method agreed within 20 per cent of the expected correlation

$$\frac{C_f}{2} = 0{\cdot}0296\, Re_x^{-0{\cdot}2}$$

for $4 \times 10^5 < Re_x < 2 \times 10^6$.

(b) Stanton numbers reported by Moffat and Kays [2, 23] agreed within 2 per cent of the correlation

$$St = 0{\cdot}0296\, Re_x^{-0{\cdot}2}\, Pr^{-0{\cdot}4}$$

for $4 \times 10^5 < Re_x < 2{\cdot}3 \times 10^6$.

(c) Mean velocity profile data taken along the centerline of the test section exhibited U^+ vs. y^+ similarity near the wall ($y^+ < 150$) [1].

(d) The deviation of the mean velocity profiles from U^+ vs. y^+ similarity in the "wake" region or outer portion near the freestream was found to be "normal" [1]. The criterion for normalcy of the "wake" region was that proposed by Coles [24] as a result of examination of nearly 500 unblown profiles.

3.2. *Mainstream conditions*

The inlet section was found to be uniform within $\pm 0{\cdot}38$ per cent in velocity and $\pm 0{\cdot}25$°F in temperature, over the entire potential flow region. A potential flow region existed for the full length of the test duct for all V_w conditions.

Using a hot wire anemometer the free-stream turbulence intensity was found to be 1·2 per cent. A spectral analysis of this fluctuation data showed substantial contributions at the mainstream blower frequency ($\sim$ 55 cps) and higher harmonics. Although the free-stream turbulence intensity level may be considered high, a comparison of the unblown U/U_∞ vs. y/δ profiles at constant Re_θ with those of Wieghardt [25], taken at a free-stream turbulence intensity of 0·25 per cent, revealed agreement within 0·01 in U/U_∞ at constant y/δ for all values of y/δ [26]. Because of this and the fact that the boundary-layer velocity profiles for the impermeable flat plate are "normal", the free-stream turbulence intensity and mean velocity variations have no noticeable effect on these profiles and are therefore considered acceptable.

3.3 *Two-dimensionality of boundary layers*

In some investigations a comparison of profiles measured at several stations across the flow have been used as a test for two-dimensionality. This test has at most a negative value [24] and fails to indicate the degree of three-dimensionality in the flow.

The more important question is how much discrepancy in experimental results arises from using the two-dimensional momentum and energy integral equations. Because of the similarity of the governing equations for flows with low velocities, constant properties, constant freestream velocity, and constant surface temperature [27], St and $C_f/2$ should both indicate any possible three-dimensional effects. On this apparatus, velocity surveys in the flow are the only means of obtaining $C_f/2$. However, St can be measured independent of any temperature survey in the boundary layer flow. For this reason, one can examine the effect of three-dimensionality on St and deduce its effect on $C_f/2$.

Three-dimensional effects are more easily detectable at high blowing conditions, since $C_f/2$ and St are very small. Hence the $V_w/U_\infty = 0{\cdot}094$ case was studied. The Stanton number St was determined by measuring the loss of thermal energy from the test plates to the flow. Whitten [3] reports that energy balance tests for $V_w/U_\infty = 0{\cdot}004$ indicate that all thermal energy can be accounted for within ± 3 per cent. For most of his heat transfer results the enthalpy thickness Reynolds numbers obtained by temperature and velocity traverses were randomly within 2–3 per cent (within experimental uncertainty) of those obtained with the two-dimensional energy integral equation by integration of St and V_w/U_∞ along the plate.

In view of these results, one must conclude that any three-dimensional effects on St, and therefore $C_f/2$, are of the order of the experimental uncertainty in Stanton Number, which is 2–3 per cent. This is consistent with the fact that the $C_f/2$ values obtained by the two-dimensional momentum integral equation are in random agreement, within stated experimental uncertainties, with the values obtained by the sublayer method, as discussed in sections 4 and 5.

3.4. *Surface conditions*

The surface roughness effects have been widely investigated in experiments without injection but nothing has been done in experiments with $V_w \neq 0$. Nikuradse [28] found that if the roughness elements were in the "viscous sublayer" $(U_\tau k_w/12\nu) \leqslant 5$ where k_w = element size) the surface was aerodynamically smooth.

Table 1. Unblown results

Date No. X stations	$Re_x \times 10^{-5}$	Re_θ	$C_f/2 \times 10^3$ Momentum integral equation	Sublayer	Best estimate
2.3.67/4	$\pm 0.25\%$	$\pm 1.9\%$	$\pm 5\%$	± 0.2	± 0.1
	4·04	1188	2·25	2·20	2·20
	9·44	2238	1·90	2·04	1·90
	12·2	2672	1·82	1·90	1·86
	17·6	3796	1·70	1·80	1·73
3.10.67/4	$\pm 0.25\%$	$\pm 1.9\%$	$\pm 5\%$	± 0.2	± 0.1
	1·31	627	2·76	2·94	2·76
	6·56	1639	2·05	2·05	2·05
	14·5	3177	1·76	1·84	1·76
	19·9	4141	1·65	1·68	1·65
7.20.67/3	$\pm 0.25\%$	$\pm 1.5\%$	$\pm 5\%$		± 0.1
	8·10	1971	1·86	—	1·96
	15·7	3352	1·74	—	1·74
	21·8	4318	1·63	—	1·63

Table 2. Uniform blowing and suction results

Date No. X stations	$\frac{\dot{m}''}{G} \times 10^3$	$Re_x \times 10^{-5}$	Re_θ	$Re_\theta - \int_0^{Re_x} \frac{\dot{m}''}{G} dRe_x$	$C_f/2 \times 10^3$ Momentum integral equation	Sublayer	Best estimate
12.28.66/7	± 0.01	$\pm 0.25\%$	$\pm 1.1\%$	$\pm 1.8\%$	$\pm 6\%$	± 0.2	± 0.1
	0·990	1·32	700	568	2·42	2·42	2·42
	0·998	3·96	1352	957	1·89	1·92	1·89
	0·993	6·65	2071	1412	1·68	1·86	1·68
	0·996	9·27	2724	1803	1·56	1·77	1·56
	0·982	12·1	3482	2297	1·47	1·63	1·47
	0·984	14·7	4086	2636	1·41	1·53	1·41
	0·984	17·4	4887	3165	1·36	1·49	1·36
12.27.66/4	± 0.07	$\pm 0.25\%$	$\pm 0.93\%$	$\pm 2.2\%$	$\pm 7\%$	± 0.3	± 0.1
	1·883	1·34	795	541	2·00	2·20	2·00
	1·916	6·75	2480	1190	1·40	1·37	1·40
	1·887	12·2	4301	1978	1·17	1·15	1·17
	1·886	17·6	6002	2641	1·04	1·03	1·04
12.23.66/4	± 0.07	$\pm 0.25\%$	$\pm 1\%$	$\pm 2.2\%$	$\pm 7\%$	± 0.3	± 0.1
	1·944	4·00	1606	833	1·55	1·55	1·55
	1·950	9·35	3318	1509	1·22	1·17	1·22
	1·896	14·9	5093	2236	1·13	1·20	1·13
	1·869	20·3	6583	2705	1·01	1·13	1·01

Date No. X stations	$\frac{\dot{m}''}{G} \times 10^3$	$Re_\theta \times 10^{-5}$	Re_θ	$Re_\theta - \int_0^{Re_x} \frac{\dot{m}''}{G} \, dRe_x$	$C_f/2 \times 10^3$ Momentum integral equation	Sublayer	Best estimate
12.20.66/4	±0·064	±0·25%	±0·85%	±4%	±12%	±0·2	±0·1
	3·864	3·98	2141	605	1·00	1·11	1·00
	3·875	9·27	4518	931	0·74	1·34?	0·74
	3·830	14·7	7130	1490	0·63	0·69	0·63
	3·834	20·1	9429	1744	0·57	0·62	0·57
12.19.66/4	±0·063	±0·25%	±0·8%	±40%	±40%	±0·4	±0·2
	7·836	3·99	3364	62	0·27	0·23	0·36
	7·817	12·0	9732	214	0·15	0·24	0·18
	7·798	17·4	13971	264	0·12	—	0·14
	7·738	20·2	15935	160?	0·11	—	0·12
12.12.66/4	±0·063	±0·25%	±0·7%	±18%	—	±0·4	±0·2
	9·504	1·36	1671	153	—	0·63	0·50
	9·482	4·13	4290	—	—	0·18	0·17
	9·405	6·93	6686	—	—	—	0·10
	9·403	9·72	9592	—	—	—	0·05
	9·346	12·5	12580	—	—	—	0·05
	9·376	15·3	14940	—	—	—	0·05
	9·479	17·9	17121	—	—	—	0·05
	9·408	20·8	20000	—	—	—	0·05
2.10.67/5	±0·01	±0·25%	±2·2%	±1·4%	±5%	±0·25	±0·1
	−1·161	3·98	895	1364	2·81	2·61	2·81
	−1·183	6·68	1238	2090	2·60	2·67	2·60
	−1·166	9·33	1595	2685	2·47	2·60	2·47
	−1·168	14·7	2325	4045	2·32	2·20	2·32
	−1·152	20·2	2940	5280	2·20	2·33	2·20
2.9.67/4	±0·066	±0·25%	±3·3%	±1%	±4%	±0·25	±0·2
	−2·391	4·00	656	1630	3·44	3·18	3·44
	−2·418	9·34	1015	3280	3·11	3·00	3·11
	−2·384	14·8	1375	4930	2·95	3·32	2·95
	−2·309	20·3	1738	6560	2·83	2·97	2·83
2.8.67/5	±0·061	±0·25%	±15%	±1%	±4%	±0·3	±0·2
	−4·613	4·01	263	2126	4·7	4·3	4·6
	−4·684	6·71	226	3361	4·6	4·5	4·6
	−4·650	9·43	191	4690	4·5	4·2	4·6
	−4·616	14·9	144	6950	4·4	4·6	4·6
	−4·611	20·2	166	9450	4·3	4·5	4·6
2.6.67/4	±0·061	±0·25%	±19%	±1%	±4%	±0·3	±0·2
	−7·542	4·05	63	3140	7·7	7·1	7·6
	−7·575	9·43	57	7310	7·6	7·4	7·6
	−7·537	14·9	57	11360	7·6	7·4	7·6
	−7·619	20·7	51	15290	7·6	8·0	7·6

Table 3. $\dot{m}''\alpha X^{-0.2}$, B = *constant results*

Date/Value of B	$c\ \frac{\text{lbm}}{\text{ft}^2\text{-sec}}$	$\frac{\dot{m}''}{G} \times 10^3$	$Re_x \times 10^{-5}$	Re_θ	$Re_\theta - \int_0^{Re_x} \frac{\dot{m}''}{G} dRe_x$	$C_f/2 \times 10^3$ Momentum integral equation	Sublayer	Best estimate
3.2.67/B = 0·73	0·0522	±0·01	±0·25%	±1·1%	±2·5%	±6%	±0·17	±0·1
		1·274	3·94	1523	912	1·74	1·70	1·74
		1·151	6·59	2214	1280	1·57	1·63	1·57
		1·071	9·24	2901	1674	1·47	1·56	1·47
		0·974	14·7	4286	2519	1·35	1·28	1·35
		0·912	20·0	5399	3128	1·26	1·36	1·26
3.8.56/B = 1·79	0·101	±0·08	±0·25%	±1%	±3·5%	±7%	±0·14	±0·1
		2·420	3·93	1890	730	1·37	1·32	1·37
		2·250	6·51	2706	938	1·25	1·25	1·25
		2·057	9·25	3673	1341	1·15	1·19	1·15
		1·968	11·9	4519	1654	1·10	1·08	1·10
		1·881	14·6	5286	1908	1·06	1·01	1·06
		1·787	19·8	6706	2577	1·00	0·93	1·00
4.5.67/B = 1·74	0·101	±0·08	±0·25%	±1%	±3·5%	±7%	±0·15	±0·1
		2·18	6·80	2826	1037	1·25	1·32	1·25
		1·86	15·0	5345	1936	1·06	1·11	1·06
4.14.67/B = 3·53	0·156	±0·07	±0·25%	±1%	±4%	±10%	±0·15	±0·1
		3·285	6·81	3484	765	0·93	0·94	0·93
		2·811	15·0	6616	1454	0·80	0·82	0·80
3.14.67/B = 3·88	0·156	±0·07	±0·25%	±1%	±4%	±10%	±0·14	±0·1
		3·739	3·95	2371	553	0·96	1·01	0·96
		3·395	6·60	3464	707	0·87	0·94	0·87
		3·170	9·20	4534	917	0·82	0·81	0·82
		2·881	14·6	6830	1606	0·75	0·76	0·75
		2·717	19·8	8696	1990	0·70	0·59	0·70
4.10.67/B = 5·92	0·202	±0·06	±0·25%	±0·8%	±6%	±12%	±0·15	±0·1
		4·39	6·84	4156	566	0·74	0·78	0·74
		3·66	15·1	7951	1144	0·62	0·63	0·62

3.16.67/B = 6·78	0·202	±0·065	±0·25%	±0·8%	±6%	±12%	±0·18	±0·1
		4·959	3·94	2880	502	0·73	0·66	0·73
		4·506	6·58	4145	532	0·66	0·72	0·66
		4·131	9·19	5456	712	0·62	0·64	0·62
		3·793	14·5	7846	1020	0·56	0·57	0·56
		3·523	19·8	10073	1281	0·52	0·36	0·52
3.21.67/B = 11·7	0·251	±0·065	±0·25%	±0·8%	±9%	±18%	±0·2	±0·2
		6·081	3·94	3253	330	0·52	0·52	0·52
		5·510	6·56	4813	370	0·47	0·49	0·47
		5·152	9·22	6366	528	0·44	0·33	0·44
		4·665	14·5	9181	769	0·40	0·31	0·40
		4·394	19·8	11661	863	0·37	0·35	0·37
1.24.67/B = −0·48	−0·56	±0·01	±0·25%	±2%	±1·5%	±5%	±0·2	±0·1
		−1·318	4·11	846	1504	2·75	2·65	2·75
		−1·128	9·57	1557	2874	2·35	2·35	2·35
		−1·022	15·0	2244	4148	2·15	2·37	2·15
		−0·933	20·4	2951	5404	2·05	2·10	2·05
1.22.67/B = −0·81	−0·134	±0·066	±0·25%	±4·2%	±1%	±4%	±0·25	±0·2
		−3·801	1·35	307	908	3·80	3·83	3·80
Run no. 1		−2·859	6·76	530	2866	3·40	3·43	3·40
		−2·499	12·2	841	4628	3·10	3·43	3·10
		−2·194	20·3	1438	7151	2·76	3·20	2·76
1.22.67/B = −0·81	−0·134	±0·061	±0·25%	±4·2%	±1%	±4%	±0·25	±0·2
		−3·110	4·08	389	1914	3·80	3·47	3·80
Run no. 2		−2·640	9·50	655	3732	3·20	3·23	3·20
		−2·415	14·9	1001	5450	2·95	3·20	2·95
1.31.67/B = −1	−0·247	±0·061	±0·25%	±15%	±1%	±4%	±0·3	±0·2
		−7·265	1·35	13	1264	6·3	6·28	6·3
		−5·846	4·06	17	2988	5·8	5·46	5·8
		−5·231	6·84	20	4564	5·2	4·91	5·2
		−4·904	9·56	21	5939	4·9	4·76	4·9
		−4·676	12·4	21	7138	4·7	4·96	4·7
		−4·316	17·7	18	9548	4·4	5·06	4·4

Table 4. $\dot{m}''\alpha X$ and $\dot{m}''\alpha X^{-\frac{1}{2}}$ results

Date	$\frac{\dot{m}''}{G} \times 10^3$	$Re_x \times 10^{-5}$	Re_θ	$Re_\theta - \int_0^{Re_x} \frac{\dot{m}''}{G}$	$C_f/2 \times 10^3$ Momentum integral	Sublayer	Best estimate
				$\dot{m}''/G = 6 \times 10^{-4}[X]$			
6.13.67	±0·01	±0·25%	±1·3%	±3%		0·1	0·1
	0·912	3·88	1235	1064	—	2·14	2·14
	2·191	9·33	2947	1922	—	1·39	1·39
	3·383	14·5	4840	2357	—	0·85	0·85
	4·651	19·6	7172	2662	—	0·32	0·50
				$\dot{m}''/G = 4{\cdot}27 \times 10^{-3}[X^{-\frac{1}{2}}]$			
6.06.67	±0·065	±0·25%	±0·7%	±3%		0·1	0·1
	3·441	3·97	2695	365	—	1·08	1·01
	3·269	9·52	4777	929	—	0·98	1·01
	1·804	14·8	6449	1542	—	1·15	1·01
	1·543	19·9	7570	1770	—	0·99	1·01

If the maximum RMS roughness value of 0·0002 in. is used for k_w, $U_\tau k_w/12\nu \approx 0{\cdot}2$ for the present unblown experiments and the surface is smooth. Other unblown experiments on this apparatus [29] with U_∞ as high as 127 ft/s indicated no roughness effects.

Only a plausibility argument can be given at the present time concerning roughness effects with blowing or sucking. It was assumed that the criterion for impermeable walls holds for permeable walls, i.e. that the roughness elements must remain in the "viscous sublayer" for the walls to be considered aerodynamically smooth. The extent of this sublayer, with blowing, can be judged by the degree with which the $C_f/2$ data derived from the sublayer method agreed with the values from the momentum integral equation: Tables 1 through 4. The centerline of the probe is approximately 0·005 in. above the surface, when the probe is touching the wall, and the velocity at that elevation agrees with the laminar expectation. It would appear that the viscous region still safely includes the roughness elements.

From another view, it is evident from previous experiments [4, 12, 16] that the dimensionless sublayer thickness y_a^+ decreases with blowing and increases with sucking. This seems reasonable since high sucking suppresses turbulence and the flow is described entirely by the laminar asymptotic suction layer relation: the sublayer thickness y_a is the boundary layer thickness. For high blowing the effects of laminar viscosity near the wall are reduced and y_a^+ becomes small.

The skin friction coefficient $C_f/2$ rapidly decreases with blowing and increases with suction. Looking at the highest suction case that was examined ($C_f/2 = 0{\cdot}0076$) we see that $U_\tau k_w/12\nu \approx 0{\cdot}5$ using $k_w = 0{\cdot}0002$ in. For the highest blowing case examined $C_f/2 \approx 0{\cdot}0001$ and $U_\tau k_w/12\nu \approx 0{\cdot}05$. In light of these very small numbers it was concluded that the test surface is aerodynamically smooth with blowing and sucking.

To prevent localized jetting of fluid to or from the boundary layer, the pore opening and pore spacing must be small such that the V_w inertia forces are small compared to the viscous forces at the surface. Since the maximum particle Reynolds number based on V_w is of order one, viscous forces govern the flow normal to the plate surface in the vicinity of the surface particles. As the injected or sucked fluid passes around the surface particles it is smoothly decelerated or accelerated, resulting in a nearly uniform V_w profile at the crests of the particles.

3.5 *Conclusion*

As a result of the positive results from the qualification tests, it was felt that the system was qualified to undertake the proposed experimental program.

4. THE EXPERIMENTAL DETERMINATION OF FRICTION FACTOR

4.1. *Momentum integral equation method*

Using the von Kármán momentum integral equation for constant free-stream velocity

$$\frac{C_f}{2} = \frac{d\theta}{dX} - \frac{\rho_w V_w}{\rho_\infty U_\infty} \tag{2}$$

and performing the indicated operations, one can obtain the friction factor. This procedure is adequate (less than ± 10 per cent uncertain) for unblown and sucked boundary layers since the percentage uncertainty in $C_f/2$ is equal to or smaller than the percentage uncertainties of either $d\theta/dX$ or $\rho_w V_w/\rho_\infty U_\infty$. However, for blown boundary layers this procedure soon fails since there is a larger percentage uncertainty in $C_f/2$ than either $d\theta/dX$ or $\rho_w V_w/\rho_\infty U_\infty$.

In the present experiments equation (2) was first integrated with respect to Re_x obtaining

$$\int_0^{Re_x} \frac{C_f}{2} d(Re_\lambda) = Re_\theta(Re_x) - Re_\theta(0) - \int_0^{Re_x} \frac{\rho_w V_w}{\rho_\infty U_\infty} d(Re_\lambda). \tag{3}$$

Since the trip was located so as to produce the virtual origin of the boundary layer at $Re_x = 0$, $Re_\theta(0)$ is zero. The right-hand side can be evaluated from integration of experimental velocity profile data and can be fitted by

$$aRe_x^d = Re_\theta(Re_x) - \int_0^{Re_x} \frac{\rho_w V_w}{\rho_\infty U_\infty} d(Re_\lambda) \tag{4}$$

where a and d are experimentally determined constants which are different for each flow condition. For flows with zero, constant, or very slowly varying V_w, unique sets of values for a and d can be determined over small Re_x ranges, such as in the present experiments. Using equations (3) and (4) one obtains

$$\int_0^{Re_x} \frac{C_f}{2} d(Re_\lambda) = aRe_x^d \tag{5}$$

Differentiating equation (5) produces

$$\frac{C_f}{2} = adRe_x^{d-1}. \tag{6}$$

There are two advantages of the second procedure over the first procedure: (a) the data information contained in the right side of equation (4) is smoothed by the curve fit before the differentiation is performed; (b) the error associated with using a single cell value of $\rho_w V_w$ tends to be eliminated when an X-averaged $\rho_w V_w$ (as detected by the flow) is used. Like the first procedure, it yields questionable friction factors as the right side of equation (4) tends to zero at high blowing rates.

4.2. *Viscous sublayer model method*

This method relies on the fact that in a thin region near the wall molecular viscosity governs the flow. Hence, neglecting X derivatives and treating the problem as laminar yields [4]

$$\frac{U}{U_\infty} = \frac{\frac{C_f}{2}}{\frac{\rho_w V_w}{\rho_\infty U_\infty}} \left[\exp\left(\frac{(\rho V)_w y}{\mu}\right) - 1\right] \tag{7}$$

from the X direction momentum equation for the flow near the wall ($U^+ < 5$ for unblown layers).

A large total pressure gradient exists near the wall and some investigators recommend corrections based on turbulence intensity, wall displacement, and velocity gradient effects. Tests conducted with the present probes in laminar asymptotic suction layers indicate that no corrections were necessary: measured profiles agreed well with the analytic profiles [1]. The turbulence correction was also ignored since the fluctuation level is low in the sublayer. Three methods of locating the probe with respect to the wall were found to yield the same results within 0·005 in [1].

A single measurement in the sublayer suffices to determine $C_f/2$ from equation (7) providing the values of $\rho_w V_w$, ρ_∞, U, U_∞, and y can accurately be measured and providing the probe is known to be in the sublayer. In many cases, several consecutive y-stations produced the same value for $C_f/2$, lending credence to the method.

4.3 *The heat transfer analogue method*

When the blowing fraction approaches 0·01 it becomes nearly impossible to measure $C_f/2$ from the hydrodynamic behavior. The momentum integral equation is dominated by the blowing term, and the dynamic pressure in the sublayer approaches zero. For these cases it is necessary to rely upon the similarity between Stanton Number and friction factor. These are generally related by some function of Prandtl Number (and possibly other terms) called the Reynolds Analogy Factor. Whitten [3] has shown that the Reynolds Analogy factor is given by

$$\frac{St}{C_f/2} \approx 1{\cdot}16 \quad \text{for} \quad V_w/U_\infty \leqslant 0{\cdot}004.$$

Assuming that this value is not markedly changed as $V_w/U_\infty \to 0{\cdot}010$ the $C_f/2$ can be determined from reported values of Stanton Number [2, 3]. It should be noted, in Table 2, that this approximation was used only for the very highest blowing, $\dot{m}''/G = 0{\cdot}0078$ and $\dot{m}''/G = 0{\cdot}0095$. This method was also used to verify the observed $C_f/2$ vs. Re_x trends for the runs reported in Table 4.

5. EXPERIMENTAL RESULTS

Skin friction coefficients are presented for constant property constant free-stream velocity flows with the following injection or suction boundary conditions:

(a) $\dot{m}'' = \text{constant}$

(b) $\dot{m}'' \alpha X^{-0{\cdot}2}$

(c) $\dot{m}'' \alpha X$

(d) $\dot{m}'' \alpha X^{-\frac{1}{2}}$.

The range of test conditions can be summarized as follows:

X-Reynolds number	$1{\cdot}3 \times 10^5 - 2 \times 10^6$
Blowing fraction, $\dot{m}''/G$	$-0{\cdot}00765 - 0{\cdot}00958$
Free-stream velocity, ft/s	42–47
Free-stream temperature, °F	64–90.

The experimental results are summarized in Figs. 2–4 and in Tables 1 through 4. Three types of $C_f/2$ values are tabulated: (1) the momentum integral equation result, (2) the sublayer method result, and (3) a "best estimate" or smoothed and most probable value for each traverse. In most cases where the momentum integral equation and the sublayer methods were both used, the exact results of the momentum integral equation were used as the best estimate. With the exception of two out of 95 velocity profile traverses, the values of $C_f/2$ for a given traverse obtained by both methods agree within the tabulated experimental uncertainty estimated at 20:1 odds using the procedure of Kline and McClintock [30], based on the instrumentation used.

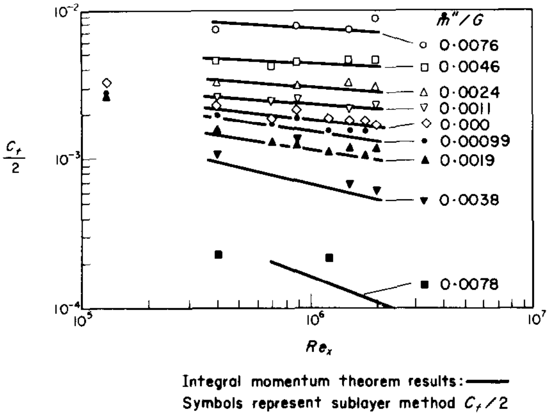

FIG. 2. $C_f/2$ vs. Re_x, $\dot{m}''$ = constant.

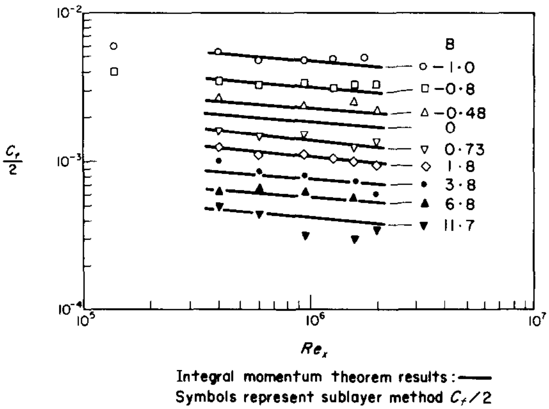

FIG. 3. $C_f/2$ vs. Re_x, $\dot{m}''/G\ \alpha X^{-0.2}$, B = constant.

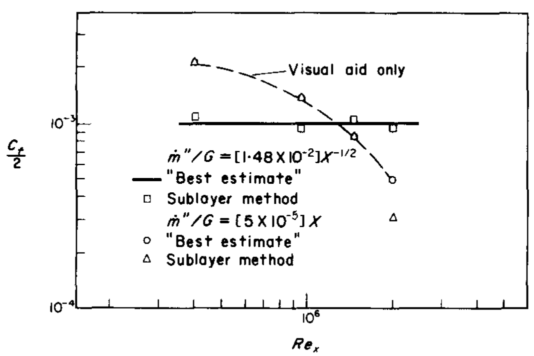

FIG. 4. $C_f/2$ vs. Re_x, $\dot{m}''\ \alpha X$ and $\dot{m}''\ \alpha X^{-\frac{1}{2}}$ runs.

The momentum integral equation, as used in this investigation, requires $C_f/2 = adRe_x^{d-1}$. It was used for all the constant $\dot{m}''$ and $\dot{m}''\alpha X^{-0.2}$ cases. (The equations describing St and $C_f/2$ are similar for constant surface temperature conditions. No compromise in the $C_f/2$ results is made by this requirement since the constant $\dot{m}''$ and $\dot{m}''\alpha X^{-0.2}$ Stanton number data taken on this apparatus [3, 23] can be easily fitted by $adRe_x^{d-1}$ over the present small Re_x range.)

For the runs with $\dot{m}''\alpha X$ and $\dot{m}''\alpha X^{-\frac{1}{2}}$, $C_f/2$ was directly determined by the viscous sublayer method only. For these runs the constant surface temperature Stanton number data of Whitten were used to verify the observed $C_f/2$ vs. Re_x trends. In the run $\dot{m}''\alpha X^{-\frac{1}{2}}$, St was constant and a constant value of $C_f/2$, obtained from the heat transfer analogue, is reported as a best estimate. The sublayer results agree randomly with this value. In the $\dot{m}''\alpha X$ run, the sublayer values are taken as best estimate values for the first three traverses while the heat transfer analogue was used for the last traverse.

In the very highest blowing cases, neither the sublayer nor the momentum integral equation method could be relied upon. In those cases the heat transfer analogue was used, with $St/C_f/2 \cong 1{\cdot}16$. In the case of high, uniform suction, asymptotic laminar profiles were observed, and analyses shows that $C_f/2$ approaches the value of $-\dot{m}''/G$. For the cases $\dot{m}''/G = -0{\cdot}0046$ and $\dot{m}''/G = -0{\cdot}0076$, the average nominal suction values were used as the reported best estimate $C_f/2$.

The present uniform injection results near $Re_x = 10^6$ are shown in Fig. 1. There is ± 10 per cent agreement among the results of Kendall, the Stevenson, Rotta, and Kinney results from the Mickley and Davis data, and the present results, with the present results having the highest values.

Consider all the present $C_f/2$ data as a whole, Assume, as others [17] have observed in turbulent boundary layer flows, that the velocity profile can be sufficiently described by local conditions in the absence of severe local disturbances. All cases of the present data must satisfy this requirement. Thus, for constant property constant free-stream velocity flow over a smooth permeable flat plate, the dimensionless relation

$$\frac{U}{U_\infty} = g\left(\frac{y}{\delta}, \frac{U_\infty \delta}{\nu}, \frac{V_w}{U_\infty}, \frac{C_f}{2}\right) \tag{8}$$

must hold [17, 31]. Using the definitions of the momentum thickness θ and the blowing parameter B, the relation

$$\frac{C_f}{2} = \frac{C_f}{2}(Re_\theta, B) \tag{9}$$

follows and must be satisfied by all cases with slowly varying $V_w(X)$.

It was observed that for $\dot{m}'' \alpha X^{-0{\cdot}2}$, B was nearly constant for each run, $d = 0{\cdot}8$, and $C_f/2 = 0{\cdot}8\ aRe_x^{-0{\cdot}2}$ where a is a function of B only. Hence for these flows, the momentum integral equation (2) can be written as

$$\frac{\mathrm{d}Re_\theta}{\mathrm{d}Re_x} = (1 + B)\left[0{\cdot}8\, aRe_x^{-0{\cdot}2}\right] \tag{10}$$

and integrated to

$$Re_\theta = a(1 + B)\, Re_x^{0{\cdot}8} \tag{11}$$

since a and B are constant for a given flow. Equation (11) can be used to eliminate Re_x to obtain

$$\frac{C_f}{2} = 0{\cdot}8\, a^{\frac{5}{4}}\, Re_\theta^{-\frac{1}{4}}(1 + B)^{\frac{1}{4}}$$

or

$$\frac{C_f}{2} = f(1 + B)\, h(Re_\theta). \tag{12}$$

Requiring $f(1) = 1$, equation (12) can be written as

$$\left.\frac{C_f/2}{C_{f_0}/2}\right|_{Re_\theta} = f(1 + B) \tag{13}$$

where $C_{f_0}/2$ is the unblown friction factor

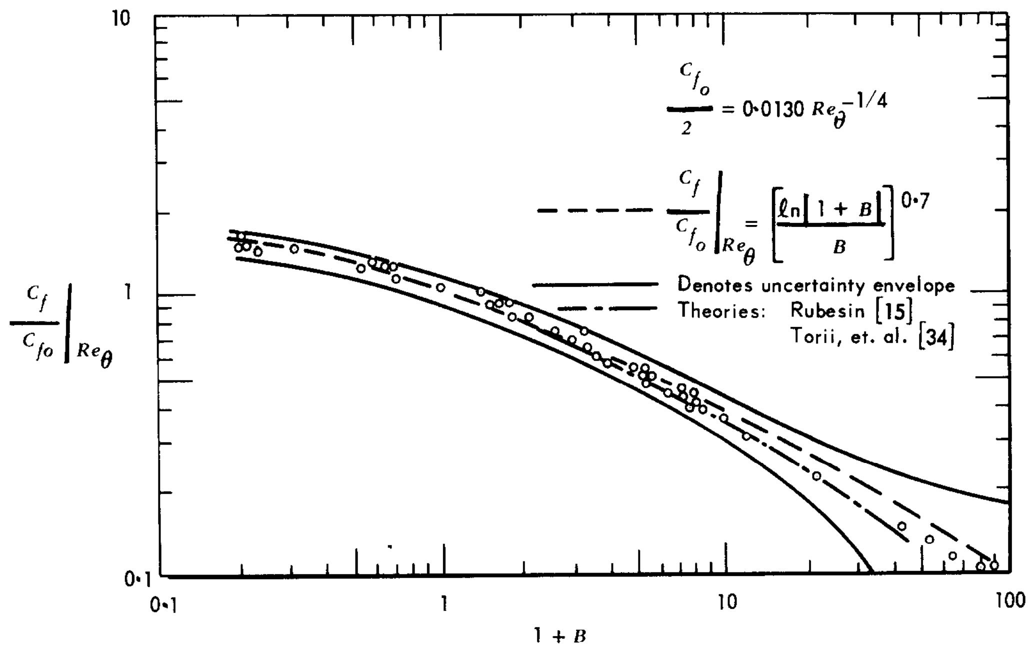

FIG. 5. $(C_f/C_{f_0})_{Re\theta}$ vs. $1 + B$, all present data, $0{\cdot}2 < 1 + B < 65$.

evaluated at the same Re_θ. Using

$$\frac{C_{f0}}{2} = 0{\cdot}0130\, Re_\theta^{-\frac{1}{4}} \tag{14}$$

which fits the present unblown $C_f/2$ data, all "best estimate" values from the turbulent flows are plotted on Fig. 5 as $C_f/C_{f0}|_{Re_\theta}$ vs. $(1 + B)$. (Since the two highest uniform suction cases and the highest $\dot{m}''\alpha X^{-0\cdot 2}$ suction case were found to have laminar-like velocity profiles, the skin friction results from those flows are not presented in Fig. 5.)

The data are found to satisfy equation (13) in addition to the necessary condition equation (4). Lines showing the uncertainty estimates envelope the data. For high blowing rates the uncertainty envelope is widened due to large uncertainties in $C_f/2$ while for turbulent flows at high sucking rates the envelope is relatively wide due to relatively large uncertainties in $(1 + B)$. However, most data fall close to the center of the uncertainty envelope.

From a practical viewpoint it is useful to fit $f(1 + B)$ by a simple empirical expression which falls within the uncertainty envelope. Following the general approach of Spalding [32], where

$$\frac{C_f}{C_{f_0}} = \frac{\ln|1 + B|}{B} \tag{15}$$

the relation of the form

$$f(1 + B) = \left[\frac{\ln|1 + B|}{B}\right]^n \tag{16}$$

where n is an empirical constant was used as a first-order approximation. As shown in Fig. 5, a good fit is obtained with $n = 0{\cdot}7$. Thus

$$\frac{C_f}{2} = 0{\cdot}0130\left[\frac{\ln|1 + B|}{B}\right]^{0\cdot 7} Re_\theta^{-\frac{1}{4}} \tag{17}$$

for the range of Re_θ examined in this study.

6. COMPARISON WITH THEORY

There are several semi-empirical theoretical solutions available for predicting the effects of blowing and suction on $C_f/2$. The theories of Rubesin [15], Kendall *et al.* [33], Torii *et al.* [34], Dorrance and Dore [35] and Stevenson [36] directly predict $C_f/2$ while the theories of Mickley *et al.* [4] and Kutateladze and Leont'ev [37] predict the change in $C_f/2$. The former group of theories are based on Prandtl mixing theory and shear stress or eddy viscosity models of the turbulent boundary layer. None of these mixing length theories result in an explicit form for the variation of $C_f/2$ with V_w/U_∞.

The Mickley theory can be described as a "stagnant film" or "Couette flow" analysis, where X-derivatives are neglected in the momentum equation and the layer thickness and transport mechanisms of the boundary layer are assumed unaffected by blowing. The resulting relationship is given by

$$\frac{C_f}{C_{f_0}} = \frac{b}{e^b - 1}. \tag{18}$$

This equation can be easily rearranged into the form of equation (15). This theory fails to indicate whether the same Re_x or Re_θ should be used in evaluating $C_{f_0}/2$.

The asymptotic theory of Kutateladze and Leontev is predicted on the existence of a limiting value of b_{crit} for which $C_f/2$ becomes zero. Analytical considerations produce $b_{\mathrm{crit}} \to 4$ as $Re_x \to \infty$, the relation

$$\left.\frac{C_f}{C_{f0}}\right|_{Re_0} = \left(1 - \frac{b}{b_{\mathrm{crit}}}\right)^2 \tag{19}$$

and the approximate solution for uniform injection and suction

$$\left.\frac{C_f}{C_{f0}}\right|_{Re_x} = \left(1 - \frac{b}{b_{\mathrm{crit}}}\right)^2\left(1 + \frac{b}{b_{\mathrm{crit}}}\right)^{-\frac{1}{4}}. \tag{20}$$

Since several of these theories were presented only for uniform injection, the present uniform injection data was used for comparison. Figure 6 presents the results from these seven theories and the present data at $Re_x \approx 10^6$. Only the theories of Rubesin and Torii *et al.* agree with all present data within the estimated uncertainty

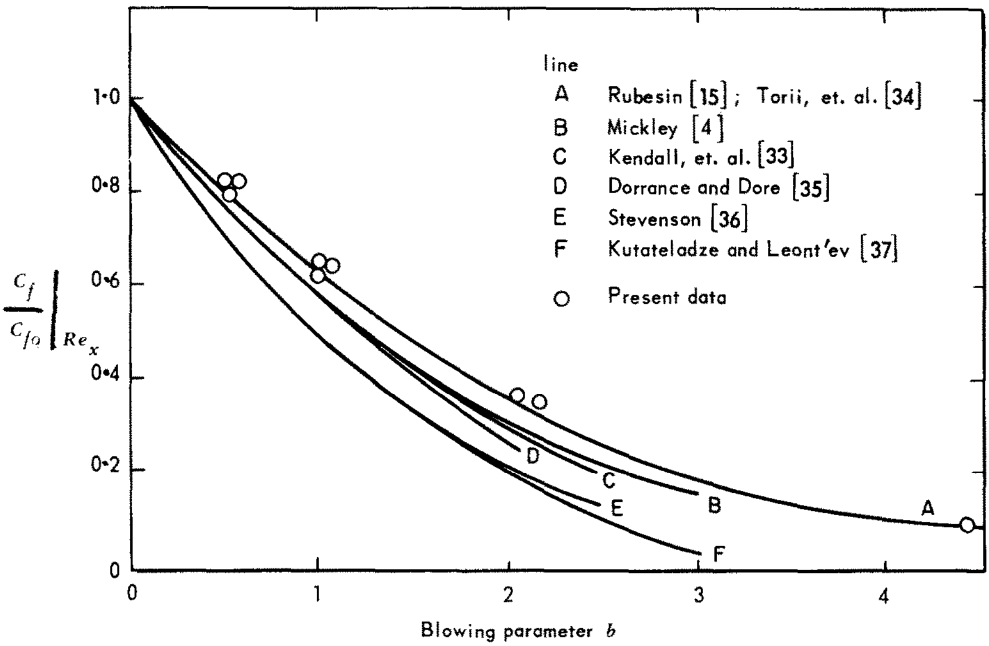

FIG. 6. Comparison of theories with data, $Re_x \approx 10^6$, uniform injection.

of the data. Comparisons at other X-Reynolds numbers produced the same conclusion.

Rubesin's theory is based on an extended law of the wall region with values in the law of the wall adjusted to give the correct $C_f/2$ when $V_w = 0$. The variation of these values with V_w is given by a sublayer thickness model selected to fit this theory to the heat-transfer data of [4]. Since Rubesin had neglected the departure of the outer region flow from the law of the wall, Torri *et al.* proposed a shear stress model for the boundary layer with injection. Numerically solving the boundary layer equations with this shear stress model and incorporating a law of the wall similar to Rubesin's, curves of $C_f/2$ and Re_θ vs. Re_x for various values of V_w/U_∞ were obtained.

In light of equations (9) and (12), the results from the theories of Rubesin and Torii *et al.* are shown in Fig. 5 for the range of Re_θ examined in the present study. Both theories closely agree with all of the present injection data within the estimated uncertainty envelope. These theories were not presented for suction. Considering the slope of the theoretical curves near $B = 0$, there is good agreement of these theories with the suction data. Hence these two thèories can be used on a local Re_θ and B basis for a theoretical description of equations (9) and (12) for slowly varying $V_w(X)$ conditions.

Thus given $V_w(X)$ for a constant property constant free-stream velocity flow, one can use equation (2) and the theories of Torii *et al.* or Rubesin on a local Re_θ and B basis to calculate $C_f/2$ vs. Re_x.

7. CONCLUSIONS

1. Experimental skin friction results have been presented for several constant and slowly varying injection and suction wall conditions for constant property, constant free-stream velocity flows. A description has been given of the flow characteristics associated with these data.
2. The present uniform injection skin friction results are in good agreement with the

results of Kendall [7] and the Stevenson [11], Rotta [12], and Kinney [13] results from the Mickley and Davis data. In light of the discussion of previous works in section 1.1., results from these three sets of data constitute a probable experimental solution to the constant free-stream velocity case with uniform injection.

3. For turbulent flows with slowly varying $V_w(X)$ along the surface
$$(|\mathrm{d}V_w/\mathrm{d}X| \ll \partial U(X,0)/\partial y),$$
$C_f/2$ was found to be a function of local Re_θ and B. This agrees with the hypothesis that turbulent boundary layers behave according to local conditions in the absence of severe local disturbances.
4. For all turbulent layers examined, $0{\cdot}2 < 1 + B < 65$, the dependence of $C_f/2$ on Re_θ and B was found to be separable, i.e. equation (13) was found to apply. An empirical correlation, equation (17), is given in this separable form.
5. Of the seven theories examined, the theories of Rubesin [15] and of Torii *et al.* [34] showed excellent agreement with all of the present injection data when considered on a local Re_θ and B basis. Although these two theories were not presented for suction, the trend for these theories near $B = 0$ indicates that there would be good agreement with the suction data.
6. A simple calculation method of $C_f/2$ vs. Re_x is now available for slowly varying $V_w(X)$. This method involves using the two-dimensional momentum integral equation and the empirical correlation, equation (17), or the theories of Torii or Rubesin on a local Re_θ and B basis.

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Résumé—Les résultats expérimentaux pour le frottement pariétal dans des couches limites avec une vitesse extérieure constante sont présentés, pour un grand nombre de conditions d'injection et d'aspiration pariétales constantes et variant lentement. On donne une description des caractéristiques de l'écoulement pour ces expériences dans l'air.

Les résultats pour l'injection uniforme sont en bon accord avec les résultats de Kandall, et les résultats de Stevenson, Rotta et Kinney obtenus à partir des données de Mickley–Davis. Pour tous les écoulements turbulents examinés, on trouve que $C_f/2$ est fonction du nombre de Reynolds local Re_θ et de B. On trouve que le rapport des coefficients de frottement $C_f/C_{f_0}|_{Re_\theta}$ est fonction seulement de B et qu'il est donné sous forme d'une fonction empirique de B.

Des sept théories examinées, les théories de Rubesin et de Torii *et al.*, sont en meilleur accord avec tous les résultats lorsqu'on les considère sur la base du nombre de Reynolds local Re et de B. On suggère un calcul simple de $C_f/2$ en fonction de Re_x pour $v_w\,(X)$ variant lentement.

Zusammenfassung—Über die Oberflächenreibung einer Grenzschricht mit konstanter Freistromgeschwindigkeit werden experimentelle Ergebnisse angegeben für eine Vielzahl von konstanten oder sich langsam ändernden Absaug- und Einblasbedingungen an der Wand. Eine Beschreibung der Strömungscharakteristika dieser Versuche mit Luft wird angegeben.

Die Ergebnisse bei einheitlichen Einblasen-stimmen gut mit den Ergebnissen von Kendall überein und mit den Ergebnissen von Stevenson, Rotta und Kinney, die auf Daten von Mickley–Davis zurückgehen. Für alle untersuchten turbulenten Strömungen wurde gefunden, dass $C_f/2$ eine Funktion der örtlichen Re-Zahl und von B ist. Der Reibungsfaktor $C_f/C_{f_0}|_{Re_\theta}$ ergibt sich als allein von B abhängig und wird als empirische Funktion von B angegeben.

Von sieben untersuchten Theorien stehen die Theorien von Rubesin, von Torii und anderen in bester Übereinstimmung mit allen Ergebnissen, wenn man sie auf eine örtliche Re-Zahl und B bezieht. Für $C_f/2$ bzw. Re_x wird eine einfache Berechnungsmethode für langsam sich änderndes $V_w\,(X)$ vorgeschlagen.

Аннотация—Приводятся экспериментальные результаты по поверхностному трению в пограничных слоях с постоянной скоростью свободного потока в условиях постоянных и медленно меняющихся скоростей отсоса и вдува. Дается описание характеристик потока воздуха в этих экспериментах.

Результаты по однородному вдуву хорошо согласуются с результатами Кенделла и других.

Для всех исследуемых турбулентных потоков найдено, что $C_f/2$ есть функция локального Re и B. Найдено, что отношение коэффициентов трения C_f/C_{f_0} Re_θ есть только функция B и дается как эмпирическая функция B.

Из семи исследуемых теорий Рубезина, Тории и других хорошо согласуются с результатами, полученными на основе локальных Re и B. Предлагается простой метод расчета $C_f/2$ в зависимости от Re_x для медленно изменяющегося $V_w(X)$.